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Natural convection in a square cavity due to thermally active plates for different boundary conditions

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ABSTRACT

This work deals with the study of natural convection cooling of thermally active plates placed inside an air filled cavity at the center, with two different boundary conditions imposed on the cavity walls. By an active plate we mean one that is hotter due to isothermal heating or inherent heat generation. The walls of the cavity are subjected to either an isothermal temperature or a uniform outward heat flux. The finite difference method using the alternating direction implicit method coupled with the successive over-relaxation technique is employed to solve the governing nonlinear coupled equations. The results are presented and discussed in terms of a steady state isotherm and streamline plot, and over all Nusselt numbers. This study will provide qualitative suggestions that may improve the thermal design of sealed modern electronic packages which are encountered frequently in the electronics industry.

1. Introduction

Natural convection cooling of compounds attached to printed circuit boards, which are placed horizontally and vertically in enclosures, is currently of great interest to the microelectronics industry, because cooling of electronic components by means of natural convection provides a simple, less expensive, reliable, maintenance free and very reliable method among the heat removal alternatives. In particular, the applications involving enclosures include electronic packages of computer components, solar collectors and energy storage systems.

A look at the literature of the present decade clearly reveals that there have been a considerable number of works published on extracting heat from hotter bodies contained in closed cavities. Natural convection in an enclosure induced by two isothermal or isoflux wall heat sources or volumetric discrete wall heat sources was investigated by Deng et al. [1]. They mainly focused on the effects of the thermal strengths of the discrete heat sources and their separation distance on the interaction between the two discrete heat sources. Oztop et al. [2] have addressed this issue with a thin heated plate built in vertically or horizontally, and found that the heat transfer is enhanced by about 20% when the plate is located vertically.

Very recently, in [3,4] the authors made an attempt to study the effects of the situation and dimension of two thin mutually orthogonal isothermally heated plates and discrete heat generating plates on the buoyancy convection in a square cavity for different positions of the plates. They found that the net heat transfer in the cavity can be enhanced by increasing the vertical plate length regardless of its position. But boundary conditions also play a vital role in industry applications. Keeping this in mind, this paper deals with heat transfer in a cavity with a thermally active plate, which is an isothermally heated plate (ITHP) in one case considered and a discretely heat generating plate (DHGP) in the other, for two different

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Nomenclature

d_i	= distance between center of cavity and baffles (m)
D_i	= dimensionless distance
g	= acceleration due to gravity (m/s^2)
Gr	= Grashof number = $g\beta\nabla TL^3/\nu_f^2$
$J(\psi, \zeta)$	= Jacobian of Ψ, ζ with respect to X_1, X_2
L	= length of the cavity (m)
Nu	= local Nusselt number = $\partial T/\partial X_i$
Nu_{wall}	= average Nusselt number = $\int_{-0.5}^{0.5} Nu dX_i$
P	= pressure (Pa)
Pr	= Prandtl number = ν_f/α_f
t	= time (s)
T	= dimensionless temperature
v_i	= velocity (m/s)
V_i	= dimensionless vertical velocity
x_i	= cartesian coordinate (m)
X_i	= dimensionless cartesian coordinate
α	= thermal diffusivity (m^2/s)
β	= volumetric coefficient of expansion of fluid (1/K)
θ	= temperature (K)
ψ	= stream function (m^2/s)
μ	= dynamic viscosity of fluid (Pa s)
ν	= kinematic viscosity = μ/ρ (m^2/s)
ρ	= density of fluid (kg/m^3)
τ	= dimensionless time
ω	= vorticity (1/s)
Ψ	= dimensionless stream function
ζ	= dimensionless vorticity.

Subscript

i	= 1, 2
c	= cold
h	= hot.

kinds of boundary conditions, namely, isothermal boundary conditions (ITBC) and isoflux boundary conditions (IFBC). This configuration model is one of the simplest cases that gives some basic information as regards designing sealed enclosures in the microelectronics industry.

2. Mathematical analysis

The physical model of the problem under consideration is an incompressible fluid filled square cavity of sides of length L with a thermally heated plate, as shown in Fig. 1. The plate is maintained at θ_h for ITHP and the plate is generating heat at a uniform rate q''' for DHGP. The plate is arbitrary placed at the center O of the cavity for two different situations, namely horizontal and vertical situations. All four cavity walls are isothermally maintained at a constant temperature θ_c which is lower than that of the thermally heated plate. Cartesian coordinates (x_1, x_2) with the corresponding velocity components (v_1, v_2) are chosen. The gravity, $\overline{g}$ acts downwards, parallel to the x_1 direction.

The nondimensional equations governing the laminar two-dimensional incompressible flow of the fluid under the Oberbeck–Boussinesq approximation [5] in the environment described above are

$$\frac{\partial \zeta}{\partial \tau} + J(\psi, \zeta) = Gr\frac{\partial T}{\partial X_2} + \nabla^2\zeta \tag{1}$$

$$\frac{\partial T_f}{\partial \tau} + J(\psi, T_f) = \frac{1}{Pr_f}\nabla^2 T_f \tag{2}$$

$$\frac{\partial T_s}{\partial \tau} = \frac{1}{Pr_f}\left[\alpha\nabla^2 T_s + \frac{1}{\rho C_p}\right] \tag{3}$$

$$\nabla^2\psi = -\zeta \tag{4}$$

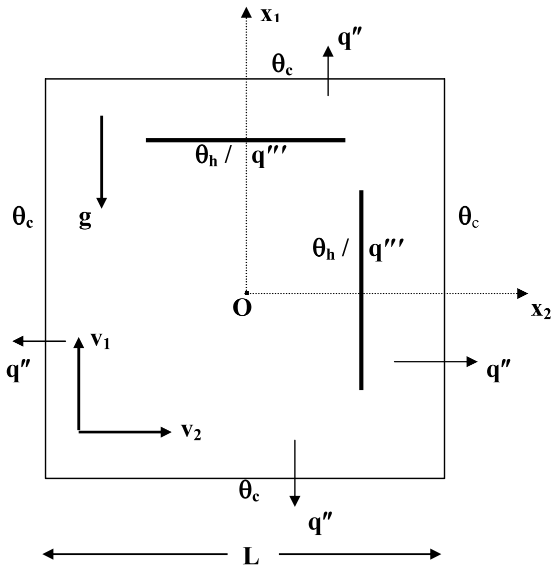


Fig. 1. Physical configuration.

with the initial and boundary conditions

$$\tau = 0; \qquad \frac{\partial \psi}{\partial X_i} = 0; \qquad T = 0; \quad \text{at } -\frac{1}{2} \le X_i \le \frac{1}{2} \tag{5}$$

$$\tau > 0; \qquad \frac{\partial \psi}{\partial X_2} = \frac{\partial \psi}{\partial X_1} = 0; \qquad T = 0; \quad \text{at } X_i = \pm\frac{1}{2} \text{ for ITBC} \tag{6}$$

$$\tau > 0; \qquad \frac{\partial \psi}{\partial X_2} = \frac{\partial \psi}{\partial X_1} = 0; \qquad \frac{\partial T}{\partial X_i} = \pm 1; \quad \text{at } X_i = \pm\frac{1}{2} \text{ for IFBC} \tag{7}$$

$$\frac{\partial \psi}{\partial X_2} = \frac{\partial \psi}{\partial X_1} = 0; \qquad T = 1 \text{ on the plates for ITHP} \tag{8}$$

$$\frac{\partial \psi}{\partial X_2} = \frac{\partial \psi}{\partial X_1} = 0; \qquad T = 1 \text{ on the plates for DHGP} \tag{9}$$

$T_f = T_s$ at fluid /plate interface for DHGP

where the subscripts $_f$ and $_s$ refer to the fluid and solid media respectively.

The dimensionless variables and parameters used in the above equations are defined as

$$X_i = \frac{x_i}{L}, \qquad D_i = \frac{d_i}{L}, \qquad \tau = \frac{t}{L^2/\nu_f}, \qquad T = \frac{\theta - \theta_c}{\Delta T}, \qquad Gr = \frac{g\beta \Delta T L^3}{\nu_f^2},$$

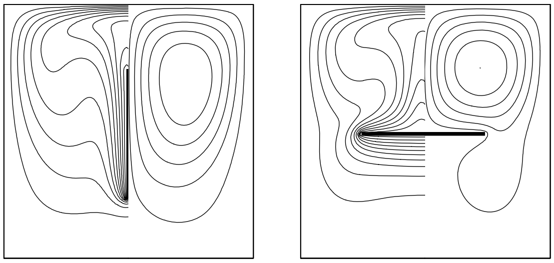
$$Pr = \frac{\nu_f}{\alpha_f}, \qquad \Delta T = \frac{q''' L^2}{\kappa_f}, \qquad \alpha = \frac{\alpha_s}{\alpha_f}, \qquad \rho C_p = \frac{(\rho C_p)_s}{(\rho C_p)_f}, \qquad \kappa = \frac{\kappa_s}{\kappa_f} \tag{10}$$

where

$$\Delta T = \begin{cases} \dfrac{1}{\theta_h - \theta_c} & \text{for ITHP} \quad \text{and} \\ \dfrac{q''' L^2}{\kappa_f} & \text{for DHGP.} \end{cases} \tag{11}$$

In order to measure the heat transfer rate in the cavity, it is necessary to define wall Nusselt numbers at the four walls: $Nu_{wall} = \int_{-0.5}^{0.5} Nu \, dX_i$, where the local Nusselt number

$$Nu = \begin{cases} \dfrac{\partial T}{\partial X_i} & \text{for ITBC} \quad \text{and} \\ \dfrac{1}{T} & \text{for IFBC.} \end{cases} \tag{12}$$



(a) $|\Psi|_{max} = 22.7903$. (b) $|\Psi|_{max} = 19.1984$.

Fig. 2. Isotherms and streamlines for ITHP with ITBC [2].

Table 1
Overall heat transfer rate for ITBC.

Position	ITHP [2]	DHGP
Horizontal	2.5152	1.8694
Vertical	3.2437	1.8793

The average Nusselt number $\overline{Nu}$ is then calculated by averaging the wall Nusselt numbers at the four walls. If one of the plates lies on a cavity wall, only the remaining part of that wall is taken into account in calculating Nu_{wall}.

In order to measure the heat transfer rate in the cavity, it is necessary to define wall Nusselt numbers at the four walls: $Nu_{wall} = \int_{-0.5}^{0.5} Nu\, dX_i$, where the local Nusselt number $Nu = \partial T/\partial X_i$. The average Nusselt number $\overline{Nu}$ is then calculated by taking the arithmetic mean of the wall Nusselt numbers at the four walls.

The motion of the liquid governed by the continuity, momentum and energy equations is obtained numerically using the finite difference method. The alternating direction implicit (ADI) technique and the successive over-relaxation (SOR) method are employed to solve the discretized equations; further details including validation of the code developed and the grid independence study can be seen in [6].

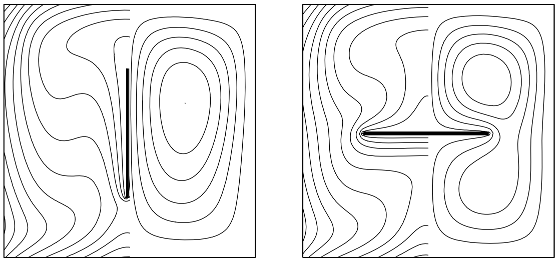
3. Results and discussion

The natural convection in an air filled square cavity induced by thermally heated plates is investigated numerically. By an active plate we mean one that is hotter due to isothermal heating or inherent heat generation. The walls of the cavity are subjected to either an isothermal temperature (ITBC) or a uniform outward heat flux (IFBC). The simulations are carried out for fixed values of $Gr = 10^6$ and $Pr = 0.71$ corresponding to air. We fix the value of the dimensionless thermal diffusivity, α, as 1, that of ρC_p as 0.0007 as in [4] and that of the thickness of the plate as in [3]. The present study deals with the heat transfer characteristics due to thermally heated plates placed at the center O of the cavity for two different situations, namely horizontal and vertical, for two kinds of boundary conditions for the cavity. Throughout the study we have plotted both the isotherms and the streamlines for ten and five equally spaced values between T_{min} and T_{max} for temperature and between zero and $|\Psi_{max}|$ for the stream function, respectively. Because the plate is at the center, the problem is symmetric about $X_2 = 0$. Hence we have plotted the streamlines and isotherms in a single plot.

Case 1. *Isothermally thermal heated plates (ITHP)*

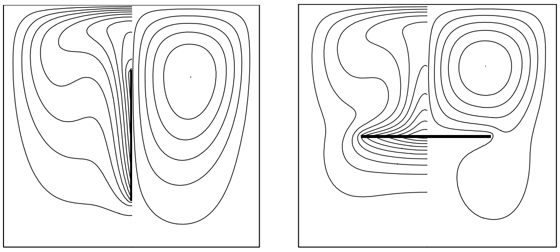
Fig. 2 shows the isotherms and streamlines when the plate is placed either vertically or horizontally at the center of the cavity for ITHP. Due to the symmetry of the boundary conditions, we observed that two counter-rotating moderate convection cells are induced at the center of the cavity. When the horizontal plate is placed at the center of the cavity, the buoyancy force is strong in the upper half of the cavity and hence two strong primary eddies are observed above the plate. The heat generating plate itself acts as a barrier to the upward flow and so convection is highly damped below the plate (see Fig. 2(a)). When the plate is placed vertically in the cavity, the resulting flow pattern is found to be bicellular, and corresponding isotherms show a strong convection. Hence this increases the overall heat transfer rate in the cavity. Hence heat transfers become more enhanced in the vertical situation than in horizontal situation. These findings are consistent with the ITHP case [3].

Fig. 3 illustrates the temperature distributions and flow patterns for IFBC when the plate is either horizontal or vertical, placed at the center of the cavity. The findings clearly indicate two counter-rotating convective cells both rising at the center of the cavity. On comparing Fig. 2 with Fig. 3, we observed that the convection is more promoted throughout the cavity for IFBC. When we look closely at Fig. 3, we find that the isotherms are more scattered throughout the cavity when the plate is placed horizontally. Hence the overall heat transfers become more enhanced for the horizontal situation than for the vertical situation (see Table 1). These results contradict the results of [2,3] for ITHP.



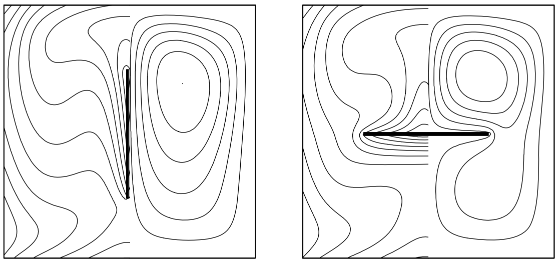
(a) $|\Psi|_{max} = 14.3615$. (b) $|\Psi|_{max} = 11.6031$.

Fig. 3. Isotherms and streamlines for ITHP with IFBC.



(a) $|\Psi|_{max} = 17.9715$. (b) $|\Psi|_{max} = 17.9729$.

Fig. 4. Isotherms and streamlines for DHGP with ITBC.



(a) $|\Psi|_{max} = 13.4364$. (b) $|\Psi|_{max} = 13.1345$.

Fig. 5. Isotherms and streamlines for DHGP with IFBC.

Table 2
Overall heat transfer rate for IFBC.

Position	ITHP	DHGP
Horizontal	2.1967	0.1173
Vertical	1.9025	0.8665

Case 2. *Discrete heated generating plates (DHGP)*

In this case, Figs. 4 and 5 show the effects of different boundary conditions for discrete heat generating plates. We observed that uninterrupted flow behavior is seen as expected. Comparing the DHGP case with the ITHP case, we observed that the strengths of the streamlines are more or less equal in the DHGP case. Hence the overall heat transfers in these cases are more or less equal for the vertical and horizontal situations for the cavity, as shown in Table 2. For the IFBC, convection is arrested below the plate for the horizontal situation. On comparing Fig. 5(b) with Fig. 3(b), we observed that the isotherms are not scattered throughout the cavity and also that the convection is arrested below the plate for the horizontal situation. Hence we observed that the overall heat transfer rate is more enhanced for the vertical situation.

4. Conclusion

Natural convection flow and heat transfer inside a square cavity due to a thermally heated plate are investigated numerically for the heated plate placed vertically or horizontally at the center of the cavity for two different kinds of boundary conditions. The study leads to the following important conclusions.

1. Heat transfer becomes more enhanced in the vertical situation than in the horizontal situation for an isothermally heated plate for isothermal boundary conditions, which is similar to the findings of previous works [2,3].
2. The overall heat transfer rate is more enhanced in the horizontal situation than in the vertical situation for isoflux boundary conditions for an isothermally heated plate. These results contradict the earlier results.
3. In the discrete heat generating case the overall heat transfer rate remains almost the same for the horizontal and vertical situations. These results contradict the earlier results for isothermally heated plates for isothermal boundary conditions.
4. In the discrete heat generating case for isoflux boundary conditions, we observed that the overall heat transfer rate is more enhanced for the vertical situation than for the horizontal situation.
5. The overall heat transfer rate and flow pattern in the cavity strongly depend on the nature of the plate and also on the boundary conditions.

Finally, this study will provide some basic ideas for designing sealed enclosures often encountered in control rooms of industrial plants.

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